NOTATION

Q, charge induced at the drop; C, equivalent capacitance; V, potential at the charging electrode; R_d, radius of charged drop; R, resistance; ρ, electrical resistivity of dispersing liquid; D_j, L_j, diameter and length of intact part of jet; D_p, diameter of aperture in plate; D_c, ring diameter; D_N, aperture diameter of attachment; q, charge density induced at jet; L_d, length of flux; $C_0 = 0.577$, Euler constant; D, distance between plates; r_j, dr_j, dz characterize the dimensions and position of the elementary cell; S_i, surface of charging electrode; φ_{ij}^{+}, φ_{ij}, two successive approximations; $R_{e\ell}$, upper-relaxation coefficient chosen empirically in the range 1.90-1.95; R_r, r_r, R_t, dimensions of ring and cylinder.

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PATH OF DROP JET AND SINGLE DROP IN ELECTRODROP-JET DEVICES

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UDC 532.51:681.327

A mathematical model describing the path of a drop jet and a single drop in all practically important deflecting systems of electrodrop-jet devices is described.

The deflection and path control of drops of working liquid are the most important processes of electrodrop-jet technology. However, there are very few engineering solutions for deflecting drop systems (DS), with practically no choice, and very precise analysis is required for the existing DS in order to ensure optimal operating conditions of the electrodrop-jet device.

The desirable complete analogy with ion–electron DS is not very useful here, since for macroparticles (drops) there arise phenomena which are not seen for electrons and ions and which significantly change the particle behavior in DS [1, 2]. In particular, because of the small ratio of the transverse dimension of the deflecting-field region and the macrodiameter, inhomogeneity of the field and edge effects play an important role where they may be disregarded in electron DS; because of the relatively small drop velocity due to the jet velocity, "cutoff" of the field is not possible here, in contrast to ion–electron DS [3]. Because $r_d \gg r_e$, aerodynamic effects that are not charcteristic of ordinary DS begin to appear, leading to new phenomena such as change in the drop acceleration, coalescence of the drops into double and triple conglomerates, etc. Therefore, the standard (for electron DS) use of series expansion in terms of the distance from the drop-flux axis in calculating the error of drop positioning is inexpedient in modeling the path in drop DS. A more informative and physically correct approach is trajectory analysis on a computer, including the solution of the field problem, integration of the equations of drop motion, and determining and optimizing the DS characteristics.

In the present work, on the basis of such analysis, a model of drop-flux motion in DS with electrodes in the form of solids of revolution, the generatrix of which L is a segment of a straight line or a second-order curve arbitrarily oriented relative to the direction of

Elektrokaplestruinaya Tekhnologiya Engineering Center, Leningrad Institute of Precision Mechanics and Optics. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 4, pp. 652-655, April, 1991. Original article submitted July 31, 1990.

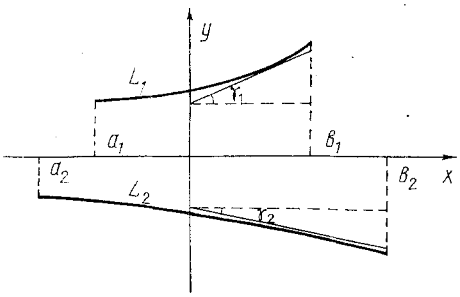

Fig. 1. Generalized calculation scheme for drop DS.

the drop path OX (Fig. 1), is constructed. The discussion covers the construction of DS with straight diverging and smoothly curved electrodes, includes the plane-parallel case, and may be generalized to the case of piecewise-discontinuous electrodes. The approximation of a plane field and small drop charges is adopted. It is assumed that the DS generatrix may be specified parametrically

$$x_{1,2} = \psi_{1,2}(\alpha, \beta), \; y_{1,2} = \chi_{1,2}(\alpha, \beta). \tag{1}$$

The field problem reduces to a boundary problem formulated as the solution of the Laplacian equation with Dirichlet boundary conditions at the DS plates with symmetry condition

$$\Delta\varphi = 0, \; \varphi|_L = \varphi_0(\overline{r}), \quad \overline{E} = -\nabla\varphi, \; \varphi(x, y, z) = -\varphi(-x, -y, z). \tag{2}$$

The solution of Eq. (2) is sought in the form of an integral representation

$$U(r) = \int_L \rho(r') K(r, r')\, dr', \tag{3}$$

where the kernel has a logarithmic singularity

$$U(r) = U(x, y) = \frac{1}{2\pi} \sum_{L_1, L_2} \int_{L_i} \rho(\alpha, \beta) \ln r ds, \quad r = \sqrt{(x-\alpha)^2 + (y-\beta)^2}. \tag{4}$$

Assuming that

$$\rho(\alpha, \beta) \approx \rho_{1,2}(\alpha, \beta) = \sum_{i=1}^{N_{1,2}} A_i \rho_i(\alpha, \beta), \tag{5}$$

the density of the charge distribution at the collocation points K, the number of which is equal to the number of unknown coefficients A_i, is determined. Then the boundary problem reduces to a system of algebraic equations

$$\sum_{i=1}^{k} B_{k,i} A_i = b_k = U(\alpha_k, \beta_k), \quad k = 1, 2, \ldots, \tag{6}$$

where

$$B_{k,i} = \int_{L_i} \rho_i(\alpha, \beta) \ln r_{\text{к}} d_s. \tag{7}$$

For the DS most often encountered in practice, with rectilinear diverging plates $\ell_{1,2} = \ell$ of slope $\gamma_{1,2} = \gamma$ = const to the plane OX, $a_{1,2} = h/2$, and deflecting potential $-U_1 = U_2 = U/2$, choosing $N_1 + 1$ and $N_2 + 1$ collocation points for electrodes 1 and 2, respectively, it is found that

$$U(x, y) \simeq \sum_{i=1}^{N_1+N_2} A_i U_i, \tag{8}$$

where

$$U_i(x, y) = \frac{1}{4\pi \cos\gamma} \int_{x_{i-1}}^{x_i} \ln \frac{(x-\alpha)^2 + \left(\alpha + \operatorname{tg}\gamma + \frac{h}{2} - y\right)^2}{(x-\alpha)^2 + \left(\alpha \operatorname{tg}\gamma + \frac{h}{2} + y\right)^2} d\alpha \tag{9}$$

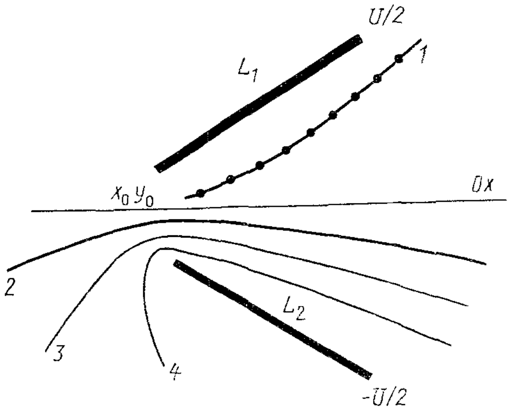

Fig. 2

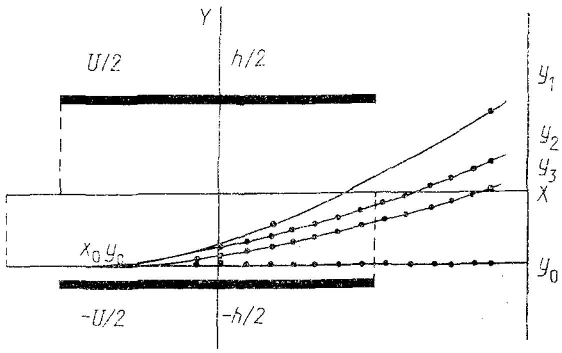

Fig. 3

Fig. 2. Trajectory of drop jet and field pattern in rectilinear divergent DS: 1) drop trajectory; 2-4) equipotential lines of field.

Fig. 3. Undeflected jet y_0, drop jet calculated in accordance with [4] y_3, and drop jet y_2 and single drop y_1 calculated by proposed model in a plane-parallel DS ($U = 8$ kV, $h = 5 \cdot 10^{-3}$ m, $\ell = 2 \cdot 10^{-2}$ m).

and

$$\overline{E}(x, y) = \sum_{i=1}^{N} E_{ix}(x, y)\,\mathbf{i} + \sum_{i=1}^{N} E_{iy}(x, y)\,\mathbf{j}. \tag{10}$$

Thus, the field problem is solved, and now the equation of drop motion must be integrated in order to determine the drop trajectory

$$m\ddot{\mathbf{r}} = q\mathbf{E}(\mathbf{r}) - k\dot{\mathbf{r}}, \quad \mathbf{r}|_{t=0} = \mathbf{r}_0, \quad \dot{\mathbf{r}}|_{t=0} = \dot{\mathbf{r}}_0, \tag{11}$$

where

$$\mathbf{r} = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}; \quad \mathbf{r}_0 = x_0\mathbf{i} + y_0\mathbf{j}; \quad \dot{\mathbf{r}}_0 = v_{x0}\mathbf{i} + v_{y0}\mathbf{j}. \tag{12}$$

The trajectory of a drop with $q/m = 0.64 \cdot 10^{-3}$ m, $v_{x0} = 12$ m/sec and the field pattern in a divergent rectilinear DS with parameters $\ell = 2 \cdot 10^{-2}$ m, $h = 5 \cdot 10^{-3}$ m, $\gamma = 35°$ according to the given model are shown in Fig. 2.

To test the correctness of the model, the results obtained are used for analysis of the trajectories of a drop jet and a single drop in a plane-parallel DS. To take correct account of the influence of aerodynamic effects here, the calculation must give a model trajectory simulating that obtained experimentally.

An apparatus [5] with a set of interchangeable emitters, electrode systems, a stroboscopic attachment, and a TV monitor, allowing the coordinate and the velocity of a drop in the jet to be recorded at any point from the DS input to the target, has been developed for experimental verification of the drop-jet path after induced capillary breakdown. A jet of Raduga ink with parameters $q/m = 0.64 \cdot 10^{-3}$. $v_{x0} = 10.95$ m/sec, $v_{y0} = 0$, $x_0 = 0.192 \cdot 10^{-3}$ m, $y_0 = -13.2 \cdot 10^{-3}$ m is modeled. The experimental drop coordinates are shown in Table 1, together with the deflection of drops which, with a spatial period y_{1e} that is 10 times greater, are assumed to be isolated; the theoretical deviations forming the model trajectory y_{jt}, y_{1t} are also shown.

It is evident from Table 1 that, for single drops, taking account of the aerodynamic drag (theoretical value K = 50) in the given model permits practically accurate simulation of the drop motion. For a drop jet of the given parameters, the aerodynamic drag is insignificant, at the level of experimental error (K = 0). However, the difference between the experimental and theoretical curves is greater than in the first case, which indicates inadequacy of the weak-charge approximation with small distances between the drops.

The trajectories of the drop jet and a single drop are shown in Fig. 3, together with the drop trajectories when no account is taken of DS edge fields and aerodynamic drag, for the sake of comparison. The theoretical deflection and the trajectories obtained using the given model for a plane-parallel DS completely coincide with the results of analytic solution in [6], which confirms that the numerical model proposed is correct.

TABLE 1. Experimental and Theoretical Drop Coordinates in Plane-Parallel DS

x_e	y_{je}	y_{1e}	y_{jt}	y_{1t}
3,3	0,063	0,1	0,069	0,10
7,6	0,35	0,4	0,36	0,40
12,0	0,76	0,96	0,79	0,91
16,3	1,38	1,60	1,37	1,58
20,7	2,1	2,5	2,12	2,45
27,2	3,45	4,1	3,42	4,12
31,5	4,25	5,1	4,32	5,10
35,8	5,18	6,25	5,21	6,25

NOTATION

r_d, drop radius; r_e, electron radius; $L_{1,2}$, generatrix of DS electrodes; (α, β), parametric coordinates of arbitrary point (x, y) on the generatrix; $\rho(x, y)$, linear charge density at electrodes; ds, differential arc length of generatrix; $\ell_{1,2}$, length of generatrix $L_{1,2}$; $\gamma_{1,2}$, slope of generatrix to axis OX; $a_{1,2}$, $b_{1,2}$, distance from generatrix to axis OX at DS input and output, respectively; U, deflecting potential; E, electric field strength in DS; q, charge of drop; m, drop mass; x_0, y_0, initial coordinates of undeflected drop; y_{je}, experimental deflection of drop in jet; y_{1e}, experimental deflection of single drop; y_{jt}, theoretical deflection of drop in jet; y_{1t}, theoretical deflection of single drop; x_e, drop coordinate along axis OX; v_{x0}, v_{y0}, initial velocity components of undeflected drop.

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